

BITSAT Solved Paper 2023 Session-II

1. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:

- (A) $2.4 \times 10^3 \text{ Am}^{-1}$
 (B) $1.2 \times 10^3 \text{ Am}^{-1}$
 (C) 1 Am^{-1}
 (D) $2.4 \times 10^{-3} \text{ Am}^{-1}$

Correct Option: B

Solution:

Given that number of turns, $n = 1200$

Current, $I = 2 \text{ A}$

Magnetic field at centre inside the solenoid is given by,

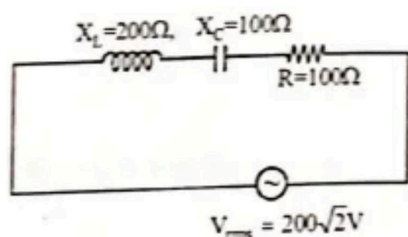
$$B = \mu_0 n I$$

So magnetic intensity at centre of the solenoid,

$$H = \frac{B}{\mu_0} = nI = \left(\frac{1200}{2}\right)(2) \quad (\because B = \mu_0 H)$$

$$H = 1.2 \times 10^3 \text{ Am}^{-1}$$

2. In the given circuit, rms value of current (I_{rms}) through the resistor R is :



- (A) 2A
 (B) $\frac{1}{2} \text{ A}$
 (C) 20A
 (D) $2\sqrt{2} \text{ A}$

Correct Option: A

Solution:

Given,

Capacitive reactance, $X_C = 100\Omega$

Inductive reactance, $X_L = 200\Omega$

Resistance, $R = 100\Omega$

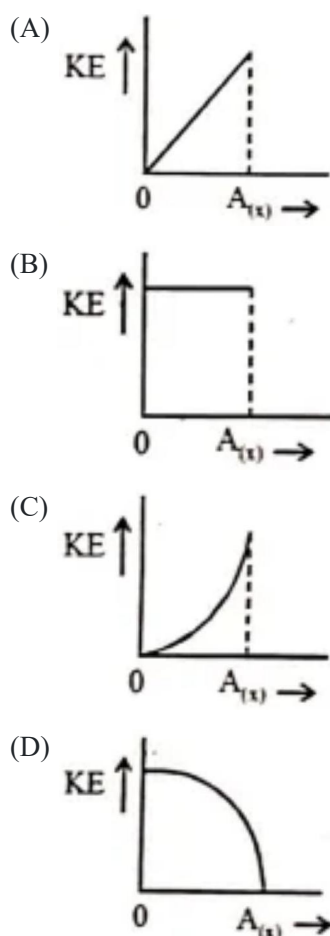
$$\text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{100^2 + (200 - 100)^2} = 100\sqrt{2}\Omega$$

RMS value of current,

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2 \text{ A}$$

3. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by



Correct Option: D

Solution:

$$T.E = K.E + P.E$$

$$\Rightarrow K.E = T.E - P.E$$

$$= \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\Rightarrow KE = \frac{1}{2}m\omega^2(A^2 - x^2) \quad [\because k = m\omega^2]$$

KE is maximum at $x = 0$ & K.E is zero when $x = A$

$\therefore$ K.E. vs x graph is parabola.

4. A projectile is projected at 30° from horizontal with initial velocity 40 ms^{-1} . The velocity of the projectile at $t = 2 \text{ s}$ from the start will be: (Given $g = 10 \text{ m/s}^2$)

(A) $20\sqrt{3} \text{ ms}^{-1}$ (B) $40\sqrt{3} \text{ ms}^{-1}$
(C) 20 ms^{-1} (D) Zero

Correct Option: A

Solution:

Given,

Initial velocity of projectile, $u = 40 \text{ m/s}$

Angle, $\theta = 30^\circ$

Time of flight

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 40 \times 1}{10 \times 2} = 4 \text{ s} \quad (\because g = 10 \text{ m/s}^2)$$

It means projectile is at maximum height at $t = 2 \text{ s}$. At maximum height vertical component of velocity is zero.

$$\begin{aligned} \text{Velocity at } t = 2 \text{ s} &= V_x = u \cos \theta = 40 \cos 30^\circ \\ &= 20\sqrt{3} \text{ ms}^{-1} \end{aligned}$$

5. On a temperature scale 'X'. The boiling point of water is 65°X and the freezing point is -15°X . Assume that the X scale is linear. The equivalent temperature corresponding to -95°X on the Fahrenheit scale would be:

(A) -63°F (B) -112°F
(C) -48°F (D) -148°F

Correct Option: D

Solution:

Given,

Boiling point of water, $= UFP = 65^\circ$

Freezing point of water, $= LFP = -15^\circ$

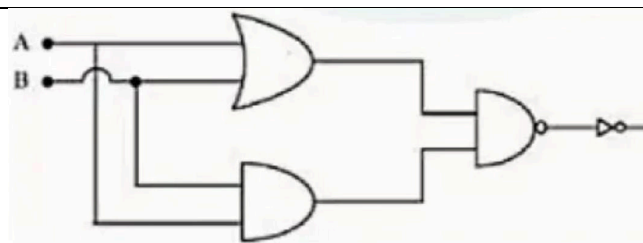
$$\frac{X - LFP}{UFP - LFP} = \frac{T_F - 32}{212 - 32}$$

$$\Rightarrow \frac{-95 - (-15)}{65 - (-15)} = \frac{T_F - 32}{180}$$

$$\Rightarrow \frac{-80}{80} = \frac{T_F - 32}{180} \Rightarrow T_F = -180 + 32$$

$$\Rightarrow T_F = -148^\circ \text{F}$$

6. The logic operations performed by the given digital circuit is equivalent to:



(A) NAND (B) NOR (C) OR (D) AND

Correct Option: D

Solution:

Step 1: Analyze the First NAND Gate The output of the first (topmost) NAND gate is: $\overline{A \cdot B}$. This output is also connected as an input to the second NAND gate (middle). Let this output be X_1 : $X_1 = \overline{A \cdot B}$. Step 2: Analyze the Second NAND Gate The second NAND gate takes X_1 and the output of the third NAND gate (X_2) as inputs. The output of the second NAND gate is: $X_3 = \overline{X_1 \cdot X_2}$. From the first NAND gate, $X_1 = \overline{A \cdot B}$, and from the third NAND gate, $X_2 = \overline{A \cdot B}$. Substituting these values: $X_3 = \overline{\overline{A \cdot B} \cdot \overline{A \cdot B}}$. Using the property of NAND gates: $X_3 = A \cdot B$. Step 3: Analyze the NOT Gate The NOT gate inverts the output of the second NAND gate. Let Y be the output of the NOT gate. Then: $Y = \overline{X_3}$. Substituting $X_3 = A \cdot B$: $Y = \overline{A \cdot B}$. Step 4: Simplify the Expression Using Boolean algebra, the final output Y simplifies to: $Y = A \cdot B$. This represents the logic for an AND gate. The given digital circuit is equivalent to an AND gate.

7. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C . How long it will take to cool from 75°C to 69°C ?

(A) 2 minutes (B) 1 minute
(C) 1.4 minutes (D) 0.5 minute

Correct Option: C

Solution:

According to Newton's Law of Cooling:

$$\frac{\Delta Q}{\Delta t} = -k(T - T_0),$$

where T_0 is the room temperature and T is the average temperature of the body during the time interval. For the first cooling phase ($98^\circ \text{C} \rightarrow 86^\circ \text{C}$):

$$\Delta t = 2 \text{ min}, \quad T_{avg} = \frac{98 + 86}{2} = 92^\circ \text{C}.$$

Substituting:



$$k = \frac{\Delta Q}{\Delta t} = \frac{1}{2} \times (92 - 22) = \frac{70}{2}.$$

For the second cooling phase ($75^\circ\text{C} \rightarrow 69^\circ\text{C}$) :

$$T_{\text{avg}} = \frac{75 + 69}{2} = 72^\circ\text{C}.$$

Substituting:

$$\Delta t = \frac{\Delta Q}{k} = \frac{1}{\frac{70}{2}} \times (72 - 22) = \frac{6}{5}.$$

Thus:

$$\Delta t = 1.4 \text{ minutes}.$$

Thus, the time taken is 1.4 minutes. The rate of cooling depends on the difference between the object's temperature and the surrounding temperature. Calculations must use the average temperature for each interval.

8. Two identical particles each of mass m go round a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be:

- (A) $\sqrt{\frac{Gm}{2a^3}}$ (B) $\sqrt{\frac{Gm}{8a^3}}$
(C) $\sqrt{\frac{Gm}{4a^3}}$ (D) $\sqrt{\frac{Gm}{a^3}}$

Correct Option: C

Solution:

Consider two identical particles, each of mass m , moving in a circle of radius a . For the particles to move in a stable circular orbit under their mutual gravitational attraction, they must be positioned diametrically opposite to each other. This ensures that the center of mass remains at the center of the circular path. Therefore, the distance between the two particles is equal to the diameter of the circle, which is $2a$.

According to Newton's law of universal gravitation, the magnitude of the mutual gravitational force of attraction between these two particles is given by the formula $F = G \frac{m_1 m_2}{r^2}$. Substituting the mass m for both particles and the distance $2a$ for r , the force becomes $F = \frac{Gm \times m}{(2a)^2}$, which simplifies to $F = \frac{Gm^2}{4a^2}$.

This gravitational force acts as the centripetal force necessary to keep each particle in its circular path.

The centripetal force for a particle of mass m moving at an angular speed ω in a circle of radius a is given by the formula $F = m\omega^2 a$.

By equating the gravitational force to the centripetal force, we establish the relationship: $\frac{Gm^2}{4a^2} = m\omega^2 a$.

Dividing both sides of the equation by m , we get:

$$\frac{Gm}{4a^2} = \omega^2 a.$$

To solve for the angular speed, we isolate ω^2 by dividing both sides by a , resulting in $\omega^2 = \frac{Gm}{4a^3}$.

Taking the square root of both sides gives the angular speed of each particle: $\omega = \sqrt{\frac{Gm}{4a^3}}$.

The angular speed of each particle is $\sqrt{\frac{Gm}{4a^3}}$.

9. Two resistances are given as $R_1 = (10 \pm 0.5)\Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurements equivalent resistances when they are connected in parallel is :

- (A) 2.33 (B) 4.33 (C) 5.33 (D) 6.33

Correct Option: B

Solution:

$$\text{Parallel} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6} \quad \frac{1}{R_{eq}} = \frac{1}{6}$$

For error :

$$\frac{dR_{eq}}{R_{eq}^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2} \Rightarrow \frac{dR_{eq}}{(6)^2} = \frac{0.5}{(10)^2} + \frac{0.5}{(15)^2} \quad dR_{eq} = 36 \times 0.5 \times \frac{13}{900} = \frac{26}{100}$$

$$\text{Now : \% error } dR_{eq} \times 100\% \text{ Req. } \frac{26/100}{6} \times 100\% \Rightarrow \frac{26}{6} = 4.33$$

10. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{1}{8}$. Then the ratio of $\frac{C_p}{C_v}$ will be

- (A) $\frac{3}{2}$ (B) $\frac{1}{2}$
(C) $\frac{4}{3}$ (D) $\frac{3}{1}$

Correct Option: C

Solution:



$$V_i = 8 \text{ litre}, V_f = 27 \text{ litre}$$

$$\frac{p_f}{p_i} = \frac{16}{81}$$

From state equation of adiabatic process

$$pV^\gamma = \text{constant}$$

or $p_i V_i^\gamma = p_f V_f^\gamma$ or $\frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma \Rightarrow \frac{16}{81} = \left(\frac{8}{27}\right)^\gamma$ or $\left(\frac{2}{3}\right)^4 = \left[\left(\frac{2}{3}\right)^3\right]^\gamma$ or $\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{3\gamma}$ On comparing both sides, we get or

$$3\gamma = 4$$

$$\gamma = \frac{4}{3} \text{ or } \frac{c_p}{c_v} = \frac{4}{3}$$

11. The torque of a force $5\hat{i} + 3\hat{j} - 7\hat{k}$ about the origin is τ . If the force acts on a particle whose position vector is $2\hat{i} + 2\hat{j} + \hat{k}$, then the value of τ will be :

- (A) $11\hat{i} + 19\hat{j} - 4\hat{k}$
 (B) $-11\hat{i} + 9\hat{j} - 16\hat{k}$
 (C) $-17\hat{i} + 19\hat{j} - 4\hat{k}$
 (D) $17\hat{i} + 9\hat{j} + 16\hat{k}$

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Correct Option: C

Solution:

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$$

$$= \hat{i}(-14 - 3) - \hat{j}(-14 - 5) + \hat{k}(6 - 10)$$

$$= -17\hat{i} + 19\hat{j} - 4\hat{k}$$

12. If $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by : (ω angular frequency) :

- (A) $\frac{1}{\omega}(\vec{K} \times \vec{E})$
 (B) $\omega(\vec{E} \times \vec{K})$
 (C) $\omega(\vec{K} \times \vec{E})$
 (D) $\vec{K} \times \vec{E}$

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Correct Option: A

Solution:

Magnetic field vector will be in the direction of $\hat{K} \times \hat{E}$

$$\text{magnitude of } B = \frac{E}{C} = \frac{K}{\omega} E$$

$$\text{Or } \vec{B} = \frac{1}{\omega}(\vec{K} \times \vec{E})$$

13. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is :

- (A) 6° (B) 1.3°
 (C) 7.8° (D) 4.5°

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Correct Option: D

Solution:

$$\delta_1 = \delta_2 \text{ [for no average deviation]}$$

$$\Rightarrow 6^\circ(1.54 - 1) = A(1.72 - 1)$$

$$\Rightarrow A = \frac{6^\circ \times 0.54}{0.72}$$

$$= \frac{18^\circ}{4} = 4.5^\circ$$

14. A flask contains hydrogen and oxygen in the ratio of 2 by mass at temperature 27°C . The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is :

- (A) 2 : 1 (B) 1 : 1 (C) 1 : 4 (D) 4 : 1

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Correct Option: B

Solution:

$$K_{av} = \frac{5}{2}kT \text{ Ratio}$$

$$= 1 : 1$$

15. The magnetic moments associated with two closely wound circular coils A and B of radius $r_A = 10 \text{ cm}$ and $r_B = 20 \text{ cm}$ respectively are equal if: (Where N_A, I_A and N_B, I_B are number of turn and current of A and B respectively)

- (A) $2 N_A I_A = N_B I_B$
 (B) $N_A = 2 N_B$
 (C) $N_A I_A = 4 N_B I_B$
 (D) $4 N_A I_A = N_B I_B$

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Correct Option: C**Solution:**

$$M = NIA$$

$$M_A = M_B$$

$$\therefore N_A I_A A_A = N_B I_B A_B$$

$$\therefore N_A I_A \pi (0.1)^2 = N_B I_B \pi (0.2)^2$$

$$\therefore N_A I_A = 4 N_B I_B$$

16. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.

(A) $\lambda_A = \lambda_B$

(B) $\lambda_A = 2\lambda_B$

(C) $\lambda_A = \lambda_B \ln 2$

(D) $\lambda_A \ln 2 = \lambda_B$

JM 2023**Correct Option: C****Solution:**

$$T_{1/2}(A) = T_{av}(B)$$

$$\frac{\ln 2}{\lambda_A} = \frac{1}{\lambda_B}$$

$$\lambda_A = \lambda_B \ln 2$$

17. The angular momentum for the electron in Bohr's orbit is L . If the electron is assumed to revolve in second orbit of hydrogen atom, then the change in angular momentum will be :

(A) $L/2$

(B) L

(C) L

(D) K

JM 2023**Correct Option: C****Solution:**

A particle of mass m moving with velocity v collides with a stationary particle of mass $2m$. After collision, they stick together and continue to move together with velocity

18. Ratio of thermal energy released in two resistor R and $3R$ connected in parallel in an electric circuit is :

(A) 3 : 1

(B) 1 : 1

(C) 1 : 3

(D) 1 : 27

JM 2023**Correct Option: A****Solution:**

$$H = \frac{V^2}{R} \times t$$

$$\frac{H_1}{H_2} = \frac{\frac{V^2 t}{R}}{\frac{V^2 t}{3R}} = 3 : 1$$

19. In a cuboid of dimension $2L \times 2L \times L$, a charge q is placed at the centre of the surface 'S' having area of $4L$. The flux through the opposite surface to 'S' is given by

(A) $\frac{q}{12\epsilon_0}$

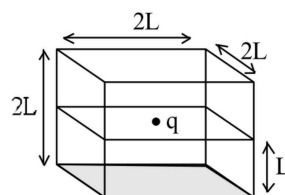
(B) $\frac{q}{3\epsilon_0}$

(C) $\frac{q}{2\epsilon_0}$

(D) $\frac{q}{6\epsilon_0}$

JM 2023**Correct Option: D****Solution:**

$$\phi = \frac{Q/\epsilon_0}{6}$$



$$\text{Flux passing through shaded face} = \frac{q}{6\epsilon_0}$$

20. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is :

(A) 4 : 3

(B) 3 : 4

(C) 16 : 9

(D) 9 : 16

JM 2023**Correct Option: D****Solution:**

$$\frac{K_1}{K_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} = \frac{m_2}{m_1} = \frac{16}{9}$$

$$\frac{m_1}{m_2} = \frac{9}{16}$$

21. The frequency (ν) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as : $\nu = r^a \rho^b s^c$. The values of a , b and c respectively are

(A) $(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2})$

(B) $(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2})$

(C) $(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2})$

(D) $(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2})$

JM 2023**Correct Option: A**

Solution:

$$[T^{-1}] = [L^1]^a [M^1 L^{-3}]^b \left[\frac{MLT^{-2}}{L}\right]^c$$

$$\Rightarrow T^{-1} = M^{b+c} \cdot L^{a-3b} \cdot T^{-2c}$$

$$c = \frac{1}{2}, b = -\frac{1}{2}, a - 3b = 0$$

$$a + \frac{3}{2} = 0 \Rightarrow a = -\frac{3}{2}$$

22. Young's moduli of the material of wires A and B are in the ratio of 1 : 4, while its area of cross sections are in the ratio of 1 : 3. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of

[Assume length of wires A and B are same]

- (A) 36 : 1 (B) 12 : 1 (C) 1 : 36 (D) 1 : 12

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Correct Option: B

Solution:

$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{A_B Y_B}{A_A Y_A} = 12$$

23. A single slit of width a is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which first minimum appears at $\theta = 30^\circ$ on the screen will be:

- (A) $0.6\mu\text{m}$ (B) $1.2\mu\text{m}$
(C) $1.8\mu\text{m}$ (D) $3\mu\text{m}$

JM 2023

Correct Option: B

Solution:

Condition for first minima is given by.

$$a \sin \theta = \lambda$$

$$a \sin 30 = 600 \times 10^{-9} \text{ m}$$

$$a = 1200 \times 10^{-9} \text{ m}$$

$$a = 1.2\mu\text{m}$$

24. A 12 V battery connected to a coil of resistance 6Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is :

- (A) 5 mH (B) 12 mH (C) 8 mH (D) 10 mH

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Correct Option: D

Solution:

$$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A}, \quad I = 0$$

[$\therefore$ switch is opened]

$$\frac{\Delta I}{\Delta t} = \frac{-2}{1 \times 10^{-3}} = -2 \times 10^3 \text{ A/sec}$$

$$e = -L \frac{\Delta I}{\Delta t}$$

$$20 = -L \times [-2 \times 10^3]$$

$$L = 0.01 \text{ H}$$

25. A passenger sitting in a train A moving at 90 km/h serves another train B moving in the opposite direction for 8 s. If the velocity of the train B is 54 km/h, the length of train B is :

- (A) 80 m (B) 200 m (C) 120 m (D) 320 m

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Correct Option: D

Solution:

Velocity of train A

$$V_A = 90 \frac{\text{km}}{\text{hr}} = 90 \times \frac{5}{18} = 25 \text{ m/s}$$

Velocity of train B

$$V_B = 54 \frac{\text{km}}{\text{hr}} = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

Velocity of train B w.r.t. train A = $\vec{V}_B - \vec{V}_A$

$$= 15 - (-25) \text{ m/s}$$

$$= 40 \text{ m/s}$$

$$\text{Time of crossing} = \frac{\text{length of train}}{\text{relative velocity}}$$

$$(8) = \frac{\ell}{40}$$

$$\ell = 8 \times 40 = 320 \text{ meter}$$

26. A proton and an α -particle are accelerated from rest by V and $4V$ potentials, respectively. The ratio of their de Broglie wavelength is:

- (A) 4 : 1 (B) 2 : 1 (C) 8 : 1 (D) 16 : 1

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Correct Option: A

Solution:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$$

$$\frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{m_p V_p q_p}{m_{\alpha} V_{\alpha} q_{\alpha}}}$$

$$\Rightarrow \frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{1 \times 2 \times 1}{4 \times 4 \times 2}} = \frac{1}{4}$$

$$\Rightarrow \lambda_p : \lambda_{\alpha} = 4 : 1$$

27. The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz . The frequency heard by a passenger inside the train is (neglect air speed. Speed of sound in air 330 ms^{-1})

(A) 200 Hz (B) 400 Hz (C) 412 Hz (D) 388 Hz

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Correct Option: B

Solution:

The relative velocity of a passenger with source of sound (engine) is 0. So there will be no doppler's effect. So frequency heard is 400 Hz .

28. If V is the gravitational potential due to sphere of uniform density on its surface, then its value at the center of sphere will be:

(A) $\frac{3V}{2}$ (B) V
(C) $\frac{4}{3}V$ (D) $\frac{V}{2}$

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Correct Option: A

Solution:

$$V = \frac{GM}{2R^3} (3R^2 - r^2) \text{ at } r = R \Rightarrow V = \left(\frac{GM}{R}\right)$$

$$\text{at } r = 0, V_0 = \frac{3GM}{2R} = \left(\frac{3}{2}V\right)$$

29. A capacitor of capacitance C is charged to a potential V . The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is:

(A) $\frac{CV}{2\epsilon_0}$
(B) $\frac{2CV}{\epsilon_0}$
(C) $\frac{CV}{\epsilon_0}$

(D) Zero

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Correct Option: C

Solution:

$$\phi = \frac{q_{in}}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0}$$

$$= \frac{CV}{\epsilon_0}$$

30. Eight equal drops of water are falling through air with a steady speed of 10 cm/s . If the drops coalesce, the new velocity is:-

(A) 10 cm/s (B) 40 cm/s
(C) 16 cm/s (D) 5 cm/s

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Correct Option: B

Solution:

$$v \propto r^2$$

$$\frac{v_1}{v_2} = \left(\frac{r}{R}\right)^2$$

$$8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$R = 2r$$

$$\frac{10}{v_2} = \left(\frac{1}{2}\right)^2$$

$$v_2 = 40 \text{ cm/s}$$

31. Identify the incorrect statement.

(A) Gangue is an ore contaminated with undesired materials.
(B) The scientific and technological process used for isolation of the metal from its ore is known as metallurgy.
(C) Minerals are naturally occurring chemical substances in the earth's crust.
(D) Ores are minerals that may contain a metal.

Correct Option: A

Solution:

Gangue is the commercially worthless material which contaminates the ore.

32. A 0.5 molal solution of ethylene glycol in water is used as coolant in a car. If the freezing point constant of water be 1.86°C per mole, the mixture shall freeze at

- (A) 0.93°C (B) -0.93°C
(C) 1.86°C (D) -1.86°C

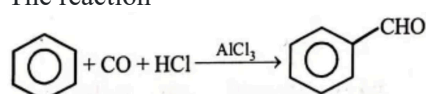
Correct Option: B

Solution:

$$\Delta T_f = K_f \times m = 1.86 \times 0.5 = 0.93^{\circ}\text{C};$$

$$T_f = -0.93^{\circ}\text{C}$$

33. The reaction



- (A) Rosenmund's reaction (B) Stephen's reaction
(C) Cannizzaro's reaction (D) Gatterman-Koch reaction

Correct Option: D

Solution:

This reaction proceeds in the presence of anhydrous AlCl_3 or CuCl .

34. Which will not result in the formation of anisole?

- (A) $\text{PhOH} + \text{CH}_3\text{Cl} \xrightarrow{\text{OH}^-}$
(B) $\text{Ph}-\text{Cl} + \text{CH}_3\text{ONa} \xrightarrow[300 \text{ atm}]{623 \text{ K}}$
(C)  $+ (\text{CH}_3)_2\text{SO}_4 \longrightarrow$
(D)  $+ \text{CH}_3\text{MgBr} \longrightarrow$

Correct Option: D

Solution:

In last case CH_4 is produced.

35. The starting material for the manufacture of KMnO_4 is

- (A) pyrolusite (B) manganite
(C) magnatite (D) haematite

Correct Option: A

Solution:

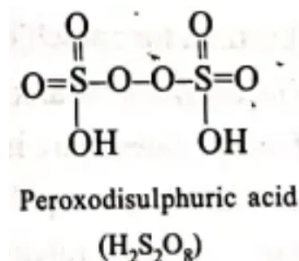
Pyrolusite (It is MnO_2)

36. Which of the following oxo acid of sulphur has O-O bond?

- (A) $\text{H}_2\text{S}_2\text{O}_7$
(B) $\text{H}_2\text{S}_2\text{O}_8$
(C) $\text{H}_2\text{S}_2\text{O}_6$
(D) $\text{H}_2\text{S}_2\text{O}_5$

Correct Option: B

Solution:

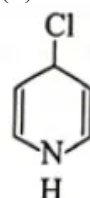


37. Identify correct reactivity order for $\text{S}_{\text{N}}1$ reaction

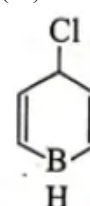
(i)



(ii)



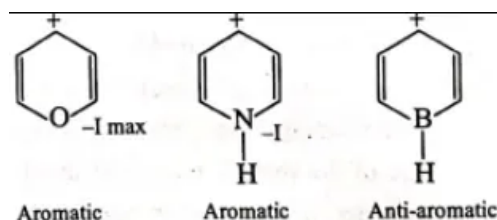
(iii)



- (A) (i) > (ii) > (iii) (B) (ii) > (iii) > (i)
(C) (i) > (iii) > (ii) (D) (ii) > (i) > (iii)

Correct Option: D

Solution:





38. Which of the following can be used to prevent the decomposition of H_2O_2 ?

- (A) Urea (B) Formaldehyde
(C) Formic acid (D) Ethanol

Correct Option: A

Solution:

Urea acts as stabilising agent. It prevents the decomposition of H_2O_2 .

39. Primary amines can be distinguished from secondary and tertiary amines by reacting with

- (A) Chloroform and alcoholic KOH (B) Methyl iodide
(C) Chloroform alone (D) Zinc dust

Correct Option: A

Solution:

1° amines (aliphatic and aromatic) react with CHCl_3/KOH to yield isocyanide (foul smelling). This is known as carbylamine test which is not given by 2° and 3° amines.

40. Which of the following is a hypnotic drug?

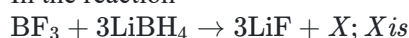
- (A) luminal (B) salol (C) catechol (D) chemisol

Correct Option: A

Solution:

These drugs induce sleep and are habit forming common example of hypnotic drugs are luminal and saconal.

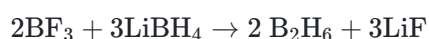
41. In the reaction



- (A) B_4H_{10} (B) B_2H_6
(C) BH_3 (D) B_3H_8

Correct Option: B

Solution:



42. Which one of the following exhibits geometrical isomerism?

- (A) 1,2-Dibromopropene (B) 2,3-Dimethylbut-2-ene
(C) 2,3-Dibromobut-2-ene (D) Both (a) and (c)

Correct Option: D

Solution:

Alkenes having double bonds with two different groups on each end of the double bond show geometrical isomerism $\text{A}_2\text{B}_2\text{C}_2$, $\text{A}_2\text{B}_2\text{CD}$, A_2BCDE .

43. Magnesium burns in CO_2 to form

- (A) $\text{MgO} + \text{C}$ (B) $\text{MgO} + \text{CO}$
(C) MgCO_3 (D) MgO

Correct Option: A

Solution:

Mg burns in CO_2 to give MgO and C .

44. Which of the following catalyst is used in preparation of high density polythene ?

- (A) Peroxide catalyst (B) Ziegler - Natta catalyst
(C) Wilkinson's catalyst (D) Pd - catalyst

Correct Option: B

Solution:

High density polythene is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in presence of catalyst such as Ziegler-Natta catalyst.

45. Which of the following acts as the stationary phase in paper chromatography ?

- (A) Water (B) Alumina
(C) Silica gel (D) None of these

Correct Option: A

Solution:

Chromatography paper contains water trapped in it, which acts as the stationary phase.

46. Point out the false statement :

- (A) The colloidal solution of a liquid in liquid is called gel
(B) Hardy Schulze rule is related with coagulation
(C) Brownian movement and Tyndall effect are shown in colloidal system
(D) Gold number is a measure of the protective power of lyophilic colloid

Correct Option: A

Solution:

Colloid of liquid in liquid is called emulsion. Colloid of liquid in solid is gel.

47. Photochemical smog formed in congested metropolitan cities mainly consists of

- (A) ozone, peroxyacetyl nitrate and NO_x
 (B) smoke, peroxyacetyl nitrate and SO_2
 (C) hydrocarbons, SO_2 and CO_2
 (D) hydrocarbons, ozone and SO_x

Correct Option: A

Solution:

Photochemical smog $\Rightarrow \text{O}_3$, PAN, NO_x .

48. In van der Waal's equation of state for a nonideal gas, the term that accounts for intermolecular forces is

- (A) $(V - b)$ (B) RT
 (C) $(P + \frac{a}{V^2})$ (D) $(RT)^{-1}$

Correct Option: C

Solution:

$(P + \frac{a}{V^2})(V - b) = RT$; Here $(P + \frac{a}{V^2})$ represents the intermolecular forces.

49. Which of the following solutions of KCl will have the highest value of specific conductance?

- (A) 1.0 N (B) 0.1 N
 (C) 1.0×10^{-2} N (D) 1.0×10^{-3} N

Correct Option: A

Solution:

The specific conductance increases with concentration. The number of ions per cm^3 increase with increase of concentration.

50. Energy required to completely separate one mole of a solid ionic compound into gaseous constituent ions is called ____.

- (A) Ionisation enthalpy (B) Electron gain enthalpy
 (C) Bond dissociation enthalpy (D) Lattice enthalpy

Correct Option: D

Solution:

Lattice enthalpy is required to completely separate one mole of a solid ionic compound into gaseous constituent ions.

51. In a solid lattice, the cation has left a lattice site and is located at an interstitial position, the lattice defect is :

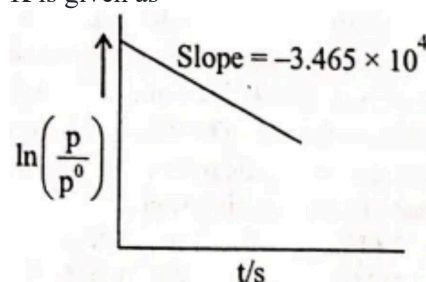
- (A) Interstitial defect (B) Valency defect
 (C) Frenkel defect (D) Schottky defect

Correct Option: C

Solution:

Frenkel defect is due to dislocation of ion from its usual lattice site to interstitial position.

52. For the decomposition of azomethane,
 $\text{CH}_3\text{N}_2\text{CH}_3(\text{g}) \rightarrow \text{CH}_3\text{CH}_3(\text{g}) + \text{N}_2(\text{g})$ a first order reaction, the variation in partial pressure with time at t K is given as



The half life of the reaction is.

- (A) 2×10^{-5} s (B) 2×10^{-4} s
 (C) 4×10^{-4} s (D) 2×10^{-3} s

Correct Option: A

Solution:

For first order reaction

$$k = \frac{1}{t} \ln \left(\frac{P_0}{P} \right) \Rightarrow \ln \left(\frac{P_0}{P} \right) = kt$$

$$\Rightarrow \ln \left(\frac{P}{P_0} \right) = -kt$$

On comparing with straight line equation $y = mx$

$$k = \text{slope} = 3.465 \times 10^4$$

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465 \times 10^4} = 2 \times 10^{-5} \text{ s}$$

53. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's orbit is r_4 . Then :

- (A) $r_4 = \frac{9}{16} r_3$ (B) $r_4 = \frac{16}{9} r_3$
 (C) $r_4 = \frac{3}{4} r_3$ (D) $r_4 = \frac{4}{3} r_3$

Correct Option: B

Solution:

$$r = 0.529 \times \frac{n^2}{Z} \text{ \AA}$$

$$r_3 = 0.529 \times \frac{3^2}{1} \Rightarrow r_4 = 0.529 \times \frac{4^2}{1}$$

$$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9} \Rightarrow r_4 = \frac{16r_3}{9}$$

54. How many elements are there in 6th period of periodic table?

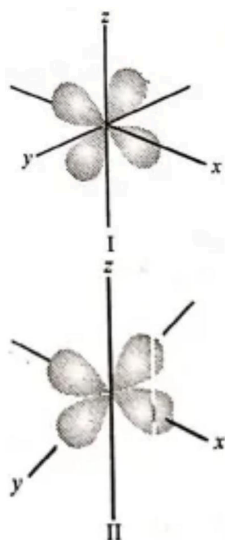
- (A) 18 (B) 8 (C) 30 (D) 32

Correct Option: D

Solution:

6th period consists of 32 elements.

55. Observe the given boundary surface diagrams of two orbitals I and II and choose the correct option.



- (A) I – $d_{x^2-y^2}$, II – d_{yz}
 (B) I – d_{yz} , II – $d_{x^2-y^2}$
 (C) I – d_{xz} , II – d_{z^2}
 (D) I – d_{xy} , II – d_{xz}

Correct Option: B

Solution:

In boundary surface diagram (1) the four lobes lie between y and z -axis (d_{yz}) whereas, in boundary surface diagram (2) the four lobes lie on the x and y -axis ($d_{x^2-y^2}$).

56. Given that $E_{O_2/H_2O}^\circ = +1.23$ V;
 $E_{S_2O_8^{2-}/SO_4^{2-}}^\circ = 2.05$ V; $E_{Br_2/Br^-}^\circ = +1.09$ V
 $E_{Au^{3+}/Au}^\circ = +1.4$ V

The strongest oxidising agent is :

- (A) Au^{3+} (B) O_2
 (C) $S_2O_8^{2-}$ (D) Br_2

Correct Option: C

Solution:

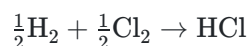
Reduction potential is maximum for $S_2O_8^{2-}$, therefore, it is the strongest oxidising agent amongst the given species

57. Given that bond energies of H – H and Cl – Cl are 430 kJ mol^{-1} and 240 kJ mol^{-1} respectively and ΔH_f° for HCl is -90 kJ mol^{-1} , bond enthalpy of HCl is

- (A) 380 kJ mol^{-1}
 (B) 425 kJ mol^{-1}
 (C) 245 kJ mol^{-1}
 (D) 290 kJ mol^{-1}

Correct Option: B

Solution:



$$\Delta H_{HCl} = \sum B.E. of reactants - \sum B.E. of products$$

$$-90 = \frac{1}{2} \times 430 + \frac{1}{2} \times 240 - B.E. of HCl$$

$$\therefore B.E. of HCl = 215 + 120 + 90$$

$$= 425 \text{ kJ mol}^{-1}$$

58. The compounds $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ constitutes a pair of

- (A) coordination isomers (B) linkage isomers
 (C) ionisation isomers (D) optical isomers

Correct Option: C

Solution:

$[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ are ionisation isomers

59. Which of the following molecules acts as a Lewis acid?

- (A) $(CH_3)_2O$
 (B) $(CH_3)_3P$
 (C) $(CH_3)_3N$
 (D) $(CH_3)_3B$

Correct Option: D

Solution:

$(CH_3)_3B$ - is an electron deficient, thus behave as a Lewis acid.



60. Thiamine and pyridoxine are also known respectively as :

- (A) Vitamin B₂ and Vitamin E
- (B) Vitamin E and Vitamin B₂
- (C) Vitamin B₆ and Vitamin B₂
- (D) Vitamin B₁ and Vitamin B₆

Correct Option: D

Solution:

Vitamine- B₁ is also known as Thiamine while vitamin B-6 is known as Pyridoxine

61. Read the given passage carefully and select the best answer to each question out of the four given alternatives.

He was a vendor of sweets. He had his own peculiar method of advertising and doing business. He never depended upon others for help and worked hard all alone. I speak of Murali- the man who sold sweets. His customers were children, the future citizens of the world. At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets. Till about eleven, the sale would be brisk. After that he moved off to other places. Even when the sweets became sticky in the heat, his business never slackened. There was depression in his business when the holidays came.

What time would he go to the school?

- (A) at 10 in the morning (B) at 8 in the morning
- (C) at 3 in the afternoon (D) at 9 in the morning

Correct Option: D

Solution:

At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets.

62. Read the given passage carefully and select the best answer to each question out of the four given alternatives.

He was a vendor of sweets. He had his own peculiar method of advertising and doing business. He never depended upon others for help and worked hard all alone. I speak of Murali- the man who sold sweets. His customers were children, the future citizens of the world. At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets. Till about eleven, the sale would be brisk. After that he moved off to other places. Even when the sweets became sticky in the heat, his business never slackened. There was depression in his business when the holidays came.

Who were his customers?

- (A) the children (B) the adults
- (C) the office goers (D) the housewives

Correct Option: A

Solution:

Murali's main customers were children.

63. Read the given passage carefully and select the best answer to each question out of the four given alternatives. He was a vendor of sweets. He had his own peculiar method of advertising and doing business. He never depended upon others for help and worked hard all alone. I speak of Murali- the man who sold sweets. His customers were children, the future citizens of the world. At the stroke of nine in the morning, Murali would stand in front of the school with his tray of sweets. Till about eleven, the sale would be brisk. After that he moved off to other places. Even when the sweets became sticky in the heat, his business never slackened. There was depression in his business when the holidays came.

Who was Murali?

- (A) a businessman (B) a vendor of sweets
- (C) a job seeker (D) a student

Correct Option: B

Solution:

Murali was a vendor of sweets.

64. In the following items, some parts of the sentence have been jumbled up. You are required to re-arrange the parts which are labelled P, Q, R, and S to produce the correct sentence. Choose the proper sequence and mark accordingly.

With six of its neighbours

P : there is a renewed warning for India

Q : and safeguard its own strategic interests

R : ranking high on global roster of failed states

S : to reassess its policy towards them

Which one of the following is the correct sequence?

- (A) P - R - S - Q (B) R - P - Q - S
- (C) P - R - Q - S (D) R - P - S - Q

Correct Option: D

Solution:

With six of its neighbours ranking high on global roster of failed states there is a renewed warning for India to reassess its policy towards them and safeguard its own strategic interests.

65. In the following items, some parts of the sentence have been jumbled up. You are required to re-arrange these parts which are labelled P, Q, R, and S to produce the correct sentence. Choose the proper sequence and mark accordingly.

A school of psychology argues that

P : is one of the manifestations of impulse control disorder, a condition in which

Q : an act harmful for oneself or others

R : motorcycling - like gambling or skydiving -

S : an individual cannot resist the impulse or temptation to perform

Which one of the following is the correct sequence?

(A) R - P - S - Q (B) Q - S - P - R

(C) R - S - P - Q (D) Q - P - S - R

Correct Option: A

Solution:

A school of psychology argues that motorcycling - like gambling or skydiving - is one of the manifestations of impulse control disorder, a condition in which an individual cannot resist the impulse or temptation to perform an act harmful for oneself or others.

66. Pick out the most effective word(s) from the given words to fill in the blank to make the sentence meaningfully complete.

Choose the correct word to fill in the blank:

We _____ in the London, but now we live in Delhi.

(A) lived (B) used to live

(C) were live (D) happen to

Correct Option: B

Solution:

The mention of 'now we live in Delhi' suggests that they lived in London in the past. So for that we use 'used to'.

67. Pick out the most effective word(s) from the given words to fill in the blank to make the sentence meaningfully complete.

Choose the correct word to complete the sentence :

She never _____ with him.

(A) agrees (B) agreeing

(C) was agreed (D) is agree

Correct Option: A

Solution:

The sentence has a general tone which is in simple past tense with the usages of verb+s because the subject is singular.

68. Pick out the opposite meaning or antonym of the word given below:

CORPULENT

(A) sallow (B) co-operative

(C) enterprising (D) emaciated

Correct Option: D

Solution:

If a person is very fat, you euphemistically call him or her corpulent. Emaciated, on the other hand, means extremely thin or weak because of illness or lack of food.

69. In the following questions, out of the four alternatives select the word similar in meaning to the word given.

Nefarious

(A) Iniquitous (B) Purposeful

(C) Suspicious (D) Virtuous

Correct Option: A

Solution:

'Nefarious' means morally bad in principles or practice.

70. In the following questions, out of the four alternatives select the word similar in meaning to the word given.

Eloquent

(A) Fluent (B) Ignorant

(C) Rude (D) Significant

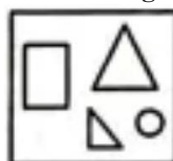
Correct Option: A

Solution:

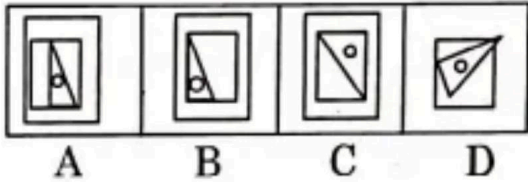
'Eloquent' means fluent or persuasive in speaking or writing.

71. Which of the answer figures is formed from the shape given the problem figure?

Problem Figure :



Answer Figure :



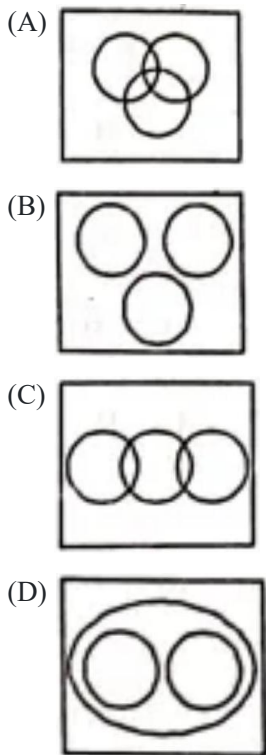
- (A) C (B) A (C) B (D) D

Correct Option: B

Solution:

It will appear like option (b), when unfolded

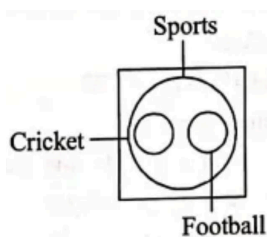
72. Select the Venn diagram that best represents the relationship between the following words Sports, Cricket, Football



Correct Option: D

Solution:

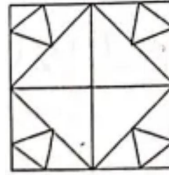
The Venn diagrams best represent the relationship between - Sports, Cricket, Football figures are shown below.



Both Cricket and Football are comes under the category of sport but cricket and football are different types of sports so they are separated here.

Hence, correct answer is option (d).

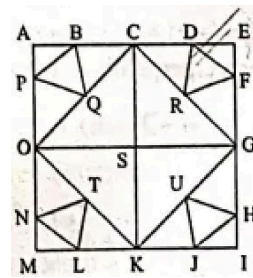
73. How many triangles are there in this figure ?



- (A) 24 (B) 26 (C) 28 (D) 20

Correct Option: C

Solution:



The triangles are:

$\triangle ABP$; $\triangle BQP$; $\triangle PQO$; $\triangle BCQ$

$\triangle ACO$; $\triangle CSO$; $\triangle CDR$; $\triangle DRF$;

$\triangle DEF$; $\triangle RFG$; $\triangle CSG$; $\triangle ECG$;

$\triangle IJH$; $\triangle JUH$; $\triangle GHU$; $\triangle JKU$

$\triangle GIH$; $\triangle SGK$; $\triangle ONT$; $\triangle NTL$;

$\triangle NML$; $\triangle TLK$; $\triangle MOK$; $\triangle SOK$;

$\triangle CGO$; $\triangle GKC$; $\triangle KGO$; $\triangle COK$

74. If ' $\div$ ' stands for 'greater than', ' $\times$ ' stands for 'addition', ' $\div$ ' stands for 'division', ' $-$ ' stands for 'equal to', ' $>$ ' stands for 'multiplication', ' $=$ ' stands for 'less than' and ' $<$ ' stands for 'minus', then which of the following alternative is correct?

- (A) $5 > 2 < 1 - 3 \times 4 \times 1$ (B) $5 < 2 \times 1 + 3 > 4 \times$
(C) $5 > 2 \times 1 - 3 > 4 < 1$ (D) $5 + 2 \times 1 = 3 + 4 >$

Correct Option: C

Solution:

$\div \rightarrow >$	$\times \rightarrow +$
$+ \rightarrow \div$	$- \rightarrow =$
$> \rightarrow \times$	$= \rightarrow <$
$< \rightarrow -$	

$$5 > 2 \times 1 - 3 > 4 < 1$$

Using the proper notations in (c), we get the statement as:

$$5 \times 2 + 1 = 3 \times 4 - 1$$

$$10 + 1 = 12 - 1$$

$$11 = 11$$

75. Find the odd one out

- (A) 37,4 (B) 24,7 (C) 42,4 (D) 14,12

Correct Option: A

Solution:

From options

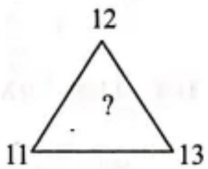
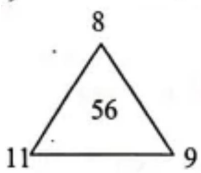
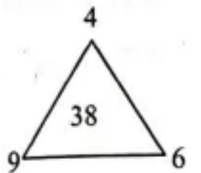
(a) $37, 4 \rightarrow 37 \times 4 = 148$ {odd one}

(b) $24, 7 \rightarrow 24 \times 7 = 168$

(c) $42, 4 \rightarrow 42 \times 4 = 168$

(d) $14, 12 \rightarrow 14 \times 12 = 168$

76. In the following question, select the number which can be placed at the sign of question mark (?) from the given alternatives.



- (A) 72 (B) 78 (C) 108 (D) 90

Correct Option: A

Solution:

As, $(4 + 6 + 9) \times 2 = 38$

$(8 + 9 + 11) \times 2 = 56$

Similarly,

$(12 + 13 + 11) \times 2 = 72$

77. Select the odd word from the given alternatives.

- (A) 2132 - 161 (B) 2678 - 672
(C) 4325 - 120 (D) 6931 - 162

Correct Option: A

Solution:

$2678 \Rightarrow 2 \times 6 \times 7 \times 8 = 672$

$4325 \Rightarrow 4 \times 3 \times 2 \times 5 = 120$

$6931 \Rightarrow 6 \times 9 \times 3 \times 1 = 162$

$2132 \Rightarrow 2 \times 1 \times 3 \times 1 = 161$

78. On the following questions, select the related numbers the attractive

$142 :: 15 :: 234 :: ?$

- (A) 45 (B) 39 (C) 33 (D) 47

Correct Option: C

Solution:

$142 : 15 \Rightarrow (1 \times 4 \times 2) + (1 + 4 + 2) = 15$

$234 : ? \Rightarrow (2 \times 3 \times 4) + (2 + 3 + 4) = 33$

79. Choose the correct alternative to complete the series. Lily, Daisy, Datura, ?

- (A) Sun Flower (B) Hibiscus
(C) Marigold (D) Jasmine

Correct Option: D

Solution:

Lily, Daisy, Datura all have outer part white and inner part yellow.

Jasmine also has outer part white and inner part yellow.

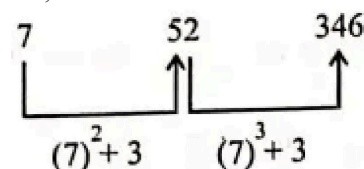
80. Select the option in which the numbers are related in same way as are the numbers of the following set. (7,52,346)

- (A) (8, 67, 515) (B) (4, 19, 70)
(C) (6, 39, 217) (D) (5, 25, 128)

Correct Option: A

Solution:

As,



Similarly,

$$\begin{array}{ccc} 8 & 67 & 515 \\ \downarrow & \uparrow & \uparrow \\ (8)^2 + 3 & (8)^3 + 3 & \end{array}$$

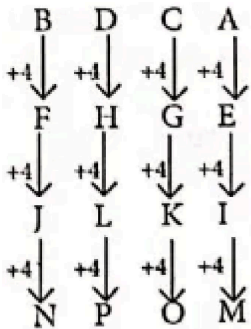
81. Find the next term of the series:

BDCA, FHGE, JLKI, ?

(A) ONMP (B) MNPO (C) NPOM (D) MONP

Correct Option: C

Solution:



82. From the given alternatives, select the word which can be formed using the letters of the given word.

REMEMBERING

(A) NEGRO (B) AGREE (C) RAIN (D) GREEN

Correct Option: D

Solution:

GREEN

83. Three statements are given, followed by three conclusions numbered I, II and III. Assuming the statements to be true, even if they seem to be at variance with commonly known facts, decide which of the conclusions logically follow(s) from the statements.

All knives are axes.

All axes are swords.

All saws are swords.

Conclusions:

I. All swords are knives.

II. Some axes are saws.

III. No axe is saw.

(A) Either conclusion II or III follows.

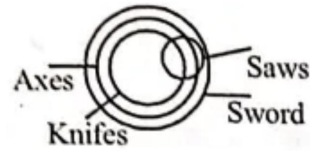
(B) Only conclusion I follows.

(C) All the conclusions follow.

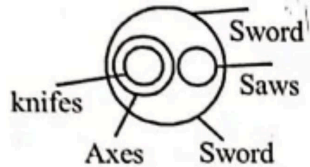
(D) Only conclusions I and II follow

Correct Option: A

Solution:



or



Hence, either 'some axes are saws' or 'no axes are saws'.

84. Arrange the following words in logical order:

1. Serve
2. Vegetable
3. Package
4. Prepare
5. Store
6. Cut

(A) 2 4 6 5 3 1

(B) 6 4 5 1 3 2

(C) 2 6 4 3 5 1

(D) 6 4 5 1 2 3

Correct Option: C

Solution:

Meaningful order of words:

2. Vegetable

↓

6. Cut

↓

4. Prepare

↓

3. Package

↓

5. Store

↓

1. Serve

85. Find the next term of the series

22, 45, 91, 183, ?

(A) 365

(B) 267

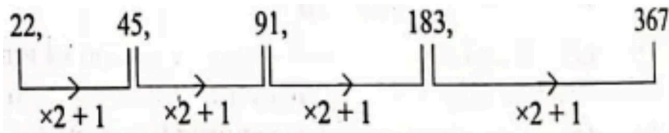
(C) 357

(D) 367

Correct Option: D

Solution:

Given Series is:



86. In a certain code language 'do re me' means 'he is late', 'fa me la' means 'she is early' and 'so ti do' means 'he leaves soon'. Which code in that language means 'late'?

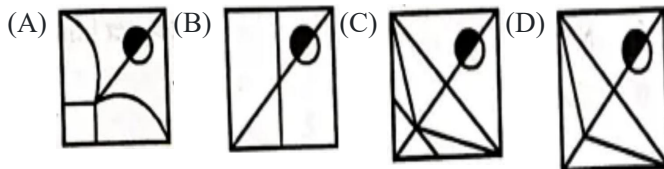
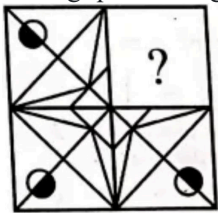
(A) la (B) do (C) me (D) re

Correct Option: D

Solution:

do re me → he is late
fa me la → she is early
so ti do → he leaves soon
Hence re → Late

87. Which answer figure will complete the pattern in the following question figure?



Correct Option: C

Solution:

Answer figure (c) is the best fit to the question figure.

88. On the following question, Select the related letters/word from the alternative.

XXIV : 48 :: XIV : ?

(A) 28 (B) 24 (C) 14 (D) 30

Correct Option: A

Solution:

XXIV × 2
⇒ 24 × 2 = 48

Similarly,

XIV × 2 = 14 × 2 = 28

89. P is father of J. S is mother of N who is brother of J. I son of S. C is sister of B. How J is related to C?

(A) Data inadequate (B) Cousin
(C) Brother (D) Sister

Correct Option: A

Solution:

P(+) ⇔ S (-)
| |
J - N(+) B(+) - C(-)

The gender of J is unknown, therefore J may be brother or sister of C.

90. In the following question, select the number which can be placed at the sign of question mark (?) from the given alternatives.

1	$\frac{1}{2}$	$\frac{3}{2}$
2	$\frac{2}{3}$	$\frac{8}{3}$
3	?	$\frac{19}{5}$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$
(C) $\frac{3}{4}$ (D) $\frac{4}{5}$

Correct Option: D

Solution:

$$\left(\frac{3}{2} - 1\right) = \frac{1}{2} \Rightarrow 1^{st} \text{ row}$$

$$\left(\frac{8}{3} - 2\right) = \frac{2}{3} \Rightarrow 2^{nd} \text{ row}$$

$$\left(\frac{19}{5} - 3\right) = \frac{4}{5} \Rightarrow 3^{rd} \text{ row}$$

91. The standard deviation of 5 scores 1, 2, 3, 4, 5 is $\sqrt{a}$. The value of 'a' is

(A) 2 (B) 3 (C) 5 (D) 1

Correct Option: A

Solution:

$$\text{Mean } (\bar{x}) = \frac{1+2+3+4+5}{5} = 3$$

$$S.D = \sigma = \sqrt{\frac{1}{5}(1 + 4 + 9 + 16 + 25) - 9} = \sqrt{11 - 9} = \sqrt{2}$$

92. Value of $\int \frac{dx}{\sqrt{x(a-x)}}$ is

(A) $2 \sin^{-1} \sqrt{\frac{x}{a}} + c$
(B) $-2 \sin^{-1} \sqrt{\frac{x}{a}} + c$



(C) $2 \sin^{-1} \frac{\sqrt{x}}{a} + c$

(D) None of these

Correct Option: A

Solution:

Let $x = a \sin^2 \theta$

then $dx = 2a \sin \theta \cos \theta d\theta$

$$\begin{aligned} \therefore I &= \int \frac{2a \sin \theta \cdot \cos \theta}{\sqrt{a \sin^2 \theta \cdot a \cos^2 \theta}} d\theta \\ &= 2 \int d\theta = 2\theta + c \\ &= 2 \sin^{-1}(\sqrt{x/a}) + c \end{aligned}$$

93. If $f(x) = \cos x$, $g(x) = \cos 2x$, $h(x) = \cos 3x$, and $I(x) = \tan x$, then which of the following options is correct?

- (A) $f(x)$ and $g(x)$ are strictly decreasing in $(0, \frac{\pi}{2})$
 (B) $h(x)$ is neither increasing nor decreasing in $(0, \frac{\pi}{2})$
 (C) $I(x)$ is strictly increasing in $(0, \frac{\pi}{2})$
 (D) All are correct

Correct Option: D

Solution:

(a) Let $f(x) = \cos x$, then $f'(x) = -\sin x$.

In interval $(0, \frac{\pi}{2})$, $f'(x) < 0$

Therefore, $f(x)$ is strictly decreasing on $(0, \frac{\pi}{2})$

(b) Let $f(x) = \cos 2x \Rightarrow f'(x) = -2 \sin 2x$

In interval $(0, \frac{\pi}{2})$, $f'(x) < 0$

Because $\sin 2x$ will either lie in the first or second quadrant which will give a positive value.

Therefore, $f(x)$ is strictly decreasing on $(0, \frac{\pi}{2})$

(c) Let $f'(x) = \cos 3x$

$\Rightarrow f'(x) = -3 \sin 3x$. In Interval $(0, \frac{\pi}{3})$, $f'(x) < 0$

Because $\sin 3x$ will either lie in the first or second quadrant which will give a positive value.

Therefore, $f(x)$ is strictly decreasing on $(0, \frac{\pi}{3})$.

When $x \in (\frac{\pi}{3}, \frac{\pi}{2})$, then $f'(x) > 0$

Because $\sin 3x$ will lie in the third quadrant.

Therefore, $f(x)$ is not strictly decreasing on $(0, \frac{\pi}{2})$

(d) Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$.

In Interval $x \in (0, \frac{\pi}{2})$, $f'(x) > 0$

Therefore, $f(x)$ is not strictly decreasing on

$(0, \frac{\pi}{2})$

94. The equation of the circle with centre $(0, 2)$ and radius is $x^2 + y^2 - my = 0$. The value of m is

- (A) 1 (B) 2 (C) 4 (D) 3

Correct Option: C

Solution:

Here $h = 0$, $k = 2$ and $r = 2$. Therefore, the required equation of the circle is :

$$(x - 0)^2 + (y - 2)^2 = (2)^2$$

$$\text{or } x^2 + y^2 - 4y + 4 = 4 \text{ or } x^2 + y^2 - 4y = 0$$

95. The modulus of $\frac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

- (A) 2 (B) 4 (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

Correct Option: D

Solution:

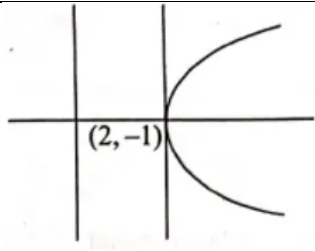
$$\begin{aligned} \frac{(1+i\sqrt{3})(2+2i)}{\sqrt{3}-i} &= \frac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i} \\ &= \frac{(2-2\sqrt{3}) + (2\sqrt{3}+2)i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} \\ &= \frac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1} \\ &= \frac{8i-8}{4} = -2+2i \\ \therefore \text{Modulus} &= \sqrt{(-2)^2 + (2)^2} = 2\sqrt{2}. \end{aligned}$$

96. If vertex of a parabola is $(2, -1)$ and the equation of its rectrix is $4x - 3y = 21$, then the length of its latus rect is

- (A) 2 (B) 8 (C) 12 (D) 16

Correct Option: B

Solution:



We are given that directrix of the parabola is $4x - 3y = 21$

and vertex of the parabola is $(2, -1)$

$$\text{Now } a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2.$$

$$\therefore \text{latus rectum of the parabola} = 4a = 8$$

97. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

(A) $\pm \frac{\sqrt{19}}{5}i$

(B) $\pm \frac{\sqrt{19}i}{2}$

(C) $\frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$

(D) $\frac{-1 \pm \sqrt{19}i}{\sqrt{5}}$

Correct Option: C

Solution:

$$\text{Here, } b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$$

Therefore, the solutions are

$$\frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}.$$

98. If x^{18} occurs in the r th term in the expansion of $(x^4 + \frac{1}{x^3})^{15}$, then what is the value of r ?

(A) 3

(B) 5

(C) 7

(D) 9

Correct Option: C

Solution:

In the expansion of $(x^4 + \frac{1}{x^3})^{15}$, let T_r is

the r^{th} term

$$\begin{aligned} T_r &= {}^{15}C_{r-1} (x^4)^{15-r+1} \left(\frac{1}{x^3}\right)^{r-1} \\ &= {}^{15}C_{r-1} x^{64-4r-3r+3} = {}^{15}C_{r-1} x^{67-7r} \end{aligned}$$

x^{18} occurs in this term

$$\Rightarrow 18 = 67 - 7r \Rightarrow 7r = 49 \Rightarrow r = 7.$$

99. $\int_0^{\pi/4} \cos x e^{\sin x} dx$ is equal to

(A) $e + 1$

(B) $e - 1$

(C) e

(D) $-e$

Correct Option: B

Solution:

$$\text{Suppose, } I = \int_0^{\pi/2} \cos x e^{\sin x} dx$$

$$\text{Let } \sin x = t \Rightarrow \cos x dx = dt$$

$$x \rightarrow 0 \Rightarrow t \rightarrow 0$$

$$\text{and } x \rightarrow \frac{\pi}{2} \Rightarrow t \rightarrow 1$$

$$\text{So } I = \int_0^1 e^t dt = [e^t]_0^1 = e - 1$$

100. The value of $\cos(2 \cos^{-1} x + \sin^{-1} x)$ at $x = \frac{1}{5}$ is

(A) $-\frac{2\sqrt{6}}{5}$

(B) $-2\sqrt{6}$

(C) $-\frac{\sqrt{6}}{5}$

(D) None of these

Correct Option: A

Solution:

$$\begin{aligned} &\cos[2 \cos^{-1} x + \sin^{-1} x] \\ &= \cos[\cos^{-1} x + \cos^{-1} x + \sin^{-1} x] \\ &= \cos[\cos^{-1} x + \pi/2] = -\sin[\cos^{-1} x] \\ &= -\sin[\sin^{-1} \sqrt{1-x^2}] = -\sqrt{1-x^2} \\ &= -\sqrt{1 - \left(\frac{1}{5}\right)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5} \end{aligned}$$

101. The shortest distance between the lines $x = y + 2 = 6z - 6$ and $x + 1 = 2y = -12z$ is

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $\frac{3}{2}$

Correct Option: B

Solution:

$$\text{The lines are } \frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$$

$$\text{and } \frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}.$$

Here,

$$\vec{a}_1 = -2\hat{j} + \hat{k}, b_1 + 6\hat{i} + 6\hat{j} + \hat{k}, \vec{a}_2 = -\hat{i},$$

$$\vec{b}_2 = 12\hat{i} + 6\hat{j} - \hat{k}$$



$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & .6 & -1 \end{vmatrix} = -12\hat{i} + 18\hat{j} - 36\hat{k}$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 - \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \frac{|(-\hat{i} + 2\hat{j} - \hat{k}) \cdot (-12\hat{i} + 18\hat{j} - 36\hat{k})|}{\sqrt{(-12)^2 + (18)^2 + (-36)^2}}$$

$$= \frac{|+12 + 36 + 36|}{\sqrt{1764}} = \frac{84}{42} = 2$$

102. If E and F are events such that $0 < P(F) < 1$, then

(A) $P(E | F) + P(\bar{E} | F) = 1$

(B) $P(E | F) + P(E | \bar{F}) = 1$

(C) $P(\bar{E} | F) + P(E | \bar{F}) = 1$

(D) $P(E | \bar{F}) + P(\bar{E} | \bar{F}) = 0$

Correct Option: A

Solution:

$$P(E | F) + P(\bar{E} | F)$$

$$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \frac{P((E \cup \bar{E}) \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

103. The integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

(A) x (B) $\log x$

(C) $\frac{1}{x}$ (D) $-x$

Correct Option: C

Solution:

$$\text{Since } x \frac{dy}{dx} - y = x^4 - 3x$$

$$\therefore \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

$$\text{Hence } IF = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

104. If $f(x) = \sqrt{1 + \cos^2(x^2)}$, then the value of $f'(\frac{\sqrt{\pi}}{2})$ is

(A) $\frac{\sqrt{\pi}}{6}$ (B) $-\sqrt{\frac{\pi}{6}}$

(C) $\frac{1}{\sqrt{6}}$ (D) $\frac{\pi}{\sqrt{6}}$

Correct Option: B

Solution:

$$\text{We have, } f(x) = \sqrt{1 + \cos^2(x^2)} \dots (i)$$

On differentiating w.r.t. x , we get

$$f'(x) = \frac{-2 \sin x^2 \cos x^2}{\sqrt{1 + \cos^2 x^2}} (x)$$

$$\Rightarrow f'(x) = \frac{-\sin 2x^2}{\sqrt{1 + \cos^2 x^2}} (x) \dots (ii)$$

Put $x = \frac{\sqrt{\pi}}{2}$ in (ii), we get

$$f'(\frac{\sqrt{\pi}}{2}) = -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin 2(\frac{\pi}{4})}{\sqrt{1 + \frac{1}{2}}}$$

$$= -\frac{\sqrt{\pi}}{2} \cdot \frac{\sin \frac{\pi}{2}}{\sqrt{\frac{3}{2}}} = -\sqrt{\frac{\pi}{6}}$$

105. The solution set of $(x-2)^{x^3-6x+8} > 1$ is

(A) $(2, \infty)$ (B) $(2, 3) \cup (4, \infty)$

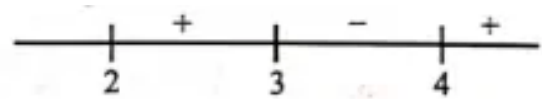
(C) $(4, 5) \cup (5, \infty)$ (D) $(2, 3) \cup (4, 5)$

Correct Option: B

Solution:

Clearly $x > 2$. Write the given inequality as

$$(x^3 - 6x + 8) \log(x-2) > 0 \Leftrightarrow (x-2)(x-4) \log(x-2) > 0 \Leftrightarrow (x-4) \log(x-2) > 0 \quad [\because x > 2]$$



106. A coin is tossed twice. Then, the probability that at least one tail occurs is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{3}{4}$

Correct Option: D

Solution:

The sample space is $S = \{HH, HT, TH, TT\}$

Let E be the event of getting at least one tail

$$\therefore E = \{HT, TH, TT\}$$

$\therefore$ Required probability p

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)} = \frac{3}{4}$$

107. If A and B are mutually exclusive events and if $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{13}{21}$, then $P(A)$ is equal to

(A) $\frac{1}{7}$ (B) $\frac{4}{7}$



(C) $\frac{2}{7}$ (D) $\frac{5}{7}$

Correct Option: C

Solution:

For mutually exclusive events

$$P(A \cup B) = P(A) + P(B) \Rightarrow P(A) = \frac{2}{7}$$

108. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are

(A) (-6, 5) (B) (5, 6) (C) (-5, 6) (D) (6, 5)

Correct Option: B

Solution:

Let (h, k) be the coordinates of the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $\frac{k-3}{h-2}$. Again the slope of the given line

$$x + y - 11 = 0 \text{ is } -1.$$

Using the condition of perpendicularity of lines, we have

$$\left(\frac{k-3}{h-2}\right)(-1) = -1 \text{ or } k - h = 1 \dots(i)$$

Since (h, k) lies on the given line, we have,

$$h + k - 11 = 0 \text{ or } h + k = 11 \dots(ii)$$

Solving (i) and (ii), we get $h = 5$ and $k = 6$. Thus (5, 6) are the required coordinates of the foot of the perpendicular.

109. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then AB is equal to

(A) B (B) A (C) O (D) I

Correct Option: C

Solution:

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac + ac \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

110. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1+\log y}{y \log y}$ (B) $\frac{(1+\log y)^2}{y \log y}$
(C) $\frac{1+\log y}{(\log y)^2}$ (D) $\frac{(1+\log y)^2}{\log y}$

Correct Option: D

Solution:

$$\text{Here, } y^x = e^{y-x}$$

Taking log on both sides, we get

$$\log y^x = \log e^{y-x}$$

$$(\because \log a^b = b \log a \text{ and } \log e = 1)$$

$$\Rightarrow x \log y = (y-x) \log e \Rightarrow x \log y = y-x \dots(i)$$

On differentiating w.r.t. x , we get

$$\frac{d}{dx}(x \log y) = \frac{d}{dx}(y-x)$$

(using product rule)

$$\Rightarrow x \left(\frac{1}{y}\right) \frac{dy}{dx} + \log y(1) = \frac{dy}{dx} - 1$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{x}{y} - 1\right) = -1 - \log y$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{y}{(1+\log y)y} - 1\right] = -(1+\log y)$$

$$[\because \text{from eq. (i), } x = \frac{y}{(1+\log y)}]$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{1-1-\log y}{1+\log y}\right] = -(1+\log y)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

111. The vertices of the hyperbola

$$9x^2 - 16y^2 - 36x + 96y - 252 = 0 \text{ are}$$

(A) (6, 3), (-2, 3) (B) (6, 3), (-6, 3)
(C) (-6, 3), (-6, -3) (D) (2, 3), (-2, 3)

Correct Option: A

Solution:

Given hyperbola is



$$\begin{aligned}
 9x^2 - 16y^2 - 36x + 96y - 252 &= 0 \\
 \Rightarrow 9(x^2 - 4x) - 16(y^2 - 6y) &= 252 \\
 \Rightarrow 9(x^2 - 4x + 4) - 16(y^2 - 6y + 9) &= 252 + 36 - 144 \\
 \Rightarrow 9(x - 2)^2 - 16(y - 3)^2 &= 144 \\
 \Rightarrow \frac{(x - 2)^2}{16} - \frac{(y - 3)^2}{9} &= 1 \quad \dots(i)
 \end{aligned}$$

Put $x = X + 2$ and $y = Y + 3$

$\therefore$ Equation (i) becomes $\frac{X^2}{16} - \frac{Y^2}{9} = 1$

Now, vertices are $X = \pm a$ where $a = 4$ and $Y = 0$

Hence, vertices are $(6, 3), (-2, 3)$.

112. Consider the function f in $A = \mathbb{R} - \{\frac{2}{3}\}$ defined as

$f(x) = \frac{4x+3}{6x-4}$, then f^{-1} is equal to :

- (A) $\frac{3+4x}{6x-4}$ (B) $\frac{6x-4}{3+4x}$
 (C) $\frac{3-4x}{6x-4}$ (D) $\frac{9+2x}{6x-4}$

Correct Option: A

Solution:

Given $f(x) = \frac{4x+3}{6x-4}$

Let $y = \frac{4x+3}{6x-4}$,

$$\begin{aligned}
 \Rightarrow 6xy - 4y &= 4x + 3 \Rightarrow x(6y - 4) = 3 + 4y \\
 \Rightarrow x &= \frac{3 + 4y}{6y - 4} \\
 f^{-1}(x) &= \frac{3 + 4x}{6x - 4}
 \end{aligned}$$

113. The sum to infinite term of the series

$$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \text{ is}$$

- (A) 3 (B) 4 (C) 6 (D) 2

Correct Option: A

Solution:

We have

$$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots \infty \quad \dots(i)$$

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots \infty \quad \dots(ii)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\begin{aligned}
 \frac{2}{3}S &= 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots \infty \\
 \Rightarrow \frac{2}{3}S &= \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots \infty \\
 \Rightarrow \frac{2}{3}S &= \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3
 \end{aligned}$$

114. The domain of the function $f(x) = \sqrt{x - \sqrt{1 - x^2}}$ is

- (A) $[-1, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, 1]$
 (B) $[-1, 1]$
 (C) $(-\infty, -\frac{1}{2}] \cup [\frac{1}{\sqrt{2}}, +\infty)$
 (D) $[\frac{1}{\sqrt{2}}, 1]$

Correct Option: D

Solution:

For $f(x)$ to be defined, we must have

$$x - \sqrt{1 - x^2} \geq 0 \quad \text{or} \quad x \geq \sqrt{1 - x^2} > 0 \quad \text{or} \quad x^2 \geq 1 - x^2$$

$$\text{or } x^2 \geq \frac{1}{2}.$$

$$\text{Now, } x^2 \geq \frac{1}{2} \Rightarrow (x - \frac{1}{\sqrt{2}})(x + \frac{1}{\sqrt{2}}) \geq 0$$

$$\Rightarrow x \leq -\frac{1}{\sqrt{2}} \quad \text{or} \quad x \geq \frac{1}{\sqrt{2}}$$

$$\text{Also, } 1 - x^2 \geq 0 \quad \text{or} \quad x^2 \leq 1.$$

$$\text{Also, } x^2 \leq 1 \Rightarrow (x - 1)(x + 1) \leq 0 \Rightarrow -1 \leq x \leq 1$$

$$\text{Thus, } x > 0, x^2 \geq \frac{1}{2} \text{ and } x^2 \leq 1 \Rightarrow x \in [\frac{1}{\sqrt{2}}, 1]$$

115. The function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing on

- (A) $(0, \infty)$ (B) $(-\infty, 0)$
 (C) $(-\infty, \infty)$ (D) None of these

Correct Option: A

Solution:

Given $f(x) = \log(1 + x) - \frac{2x}{2+x}$

$$\begin{aligned}
 f'(x) &= \frac{1}{1+x} - \frac{(2+x)(2) - 2x}{(2+x)^2} \\
 &= \frac{1}{1+x} - \frac{4}{(2+x)^2} \\
 &= \frac{(2+x)^2 - 4 - 4x}{(1+x)(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2} > 0 \text{ for all } x
 \end{aligned}$$



Thus, given function $f(x)$ is increasing on $(0, \infty)$.

116. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is

- (A) $x + 3y = 21$ (B) $2x - 3y = 7$
(C) $x + 7y = 31$ (D) $2x + 3y = 21$

Correct Option: C

Solution:

One vertex of square is $(-4, 5)$ and equation of one diagonal is $7x - y + 8 = 0$.

Diagonals of a square are perpendicular and bisect each other

Let the equation of the other diagonal be $y = mx + c$ where m is the slope and c the y-intercept.

Since this line passes through $(-4, 5)$

$$\therefore 5 = -4m + c \quad \dots(i)$$

Since this line is at right angle to the line

$$7x - y + 8 = 0 \text{ or } y = 7x + 8, \text{ having slope} = 7,$$

$$\therefore 7 \times m = -1 \text{ or } m = \frac{-1}{7}$$

Putting this value of m in equation (i) we get

$$5 = -4 \times \left(\frac{-1}{7}\right) + c$$

$$\text{or } 5 = \frac{4}{7} + c \quad \text{or } c = 5 - \frac{4}{7} = \frac{31}{7}$$

Hence equation of the other diagonal is

$$y = -\frac{1}{7}x + \frac{31}{7} \text{ or } 7y = -x + 31 \\ \text{or } x + 7y - 31 = 0 \text{ or } x + 7y = 31.$$

117. $\int \tan^{-1} \sqrt{x} \, dx$ is equal to

- (A) $(x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + C$
(B) $x \tan^{-1} \sqrt{x} - \sqrt{x} + C$
(C) $\sqrt{x} - x \tan^{-1} \sqrt{x} + C$
(D) $\sqrt{x} - (x + 1) \tan^{-1} \sqrt{x} + C$

Correct Option: A

Solution:

$$\text{We have, } I = \int 1 \cdot \tan^{-1} \sqrt{x} \, dx$$

Using by parts,

$$\begin{aligned} I &= \tan^{-1} \sqrt{x} \cdot (x) - \int \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \times x \, dx \\ &= x \tan^{-1} \sqrt{x} - \int \frac{x}{(1+x)2\sqrt{x}} \, dx \\ &= x \tan^{-1} \sqrt{x} - \int \left(\frac{1+x}{(1+x)2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}} \right) dx \\ &= x \tan^{-1} \sqrt{x} - \int \frac{dx}{2\sqrt{x}} + \int \frac{dx}{2\sqrt{x}(1+x)} \\ &= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + C \\ &= (x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + C \end{aligned}$$

118. If n is a positive integer, then $2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by:

- (A) 2 (B) 7 (C) 11 (D) 27

Correct Option: C

Solution:

$$\text{Let } P(n) = 2 \cdot 4^{2n+1} + 3^{3n+1}.$$

Then $P(1) = 2 \cdot 4^3 + 3^4 = 209$, which is divisible by 11 but not divisible by 2, 7 or 27.

Further, let $P(k) = 2 \cdot 4^{2k+1} + 3^{3k+1}$ is divisible by 11, that is, $2 \cdot 4^{2k+1} + 3^{3k+1} = 11q$ for some integer q . Now

$$\begin{aligned} P(k+1) &= 2 \cdot 4^{2k+3} + 3^{3k+4} \\ &= 2 \cdot 4^{2k+1} \cdot 4^2 + 3^{3k+1} \cdot 3^3 \\ &= 16 \cdot 2 \cdot 4^{2k+1} + 27 \cdot 3^{3k+1} \\ &= 16 \cdot 2 \cdot 4^{2k+1} + (16 + 11) \cdot 3^{3k+1} \\ &= 16 [2 \cdot 4^{2k+1} + 3^{3k+1}] + 11 \cdot 3^{3k+1} \\ &= 16 \cdot 11q + 11 \cdot 3^{3k+1} \\ &= 11 (16q + 3^{3k+1}) = 11m \end{aligned}$$

where $m = 16q + 3^{3k+1}$ is another integer.

$\therefore P(k+1)$ is divisible by 11.

119. If $n(A) = 1000$, $n(B) = 500$ and if $n(A \cap B) \geq 1$ and $n(A \cup B) = p$, then

- (A) $500 \leq p \leq 1000$ (B) $1001 \leq p \leq 1498$
(C) $1000 \leq p \leq 1498$ (D) $1000 \leq p \leq 1499$

Correct Option: D

Solution:

$$n(A) = 1000, n(B) = 500, n(A \cap B) \geq 1,$$

$$n(A \cup B) = p; n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$p = 1000 + 500 - n(A \cap B) \Rightarrow 1 \leq n(A \cap B) \leq 500$$

$$\text{Hence } p \leq 1499 \text{ and } p \geq 1000 \Rightarrow 1000 \leq p \leq 1499$$

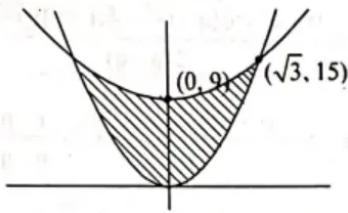
120. The area of the region

$$R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\} \text{ is:}$$

- (A) $11\sqrt{3}$ square units (B) $12\sqrt{3}$ square units
(C) $9\sqrt{3}$ square units (D) $6\sqrt{3}$ square units

Correct Option: B

Solution:



$$\text{Required area} = 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx$$

$$= 2 [9x - x^3]_0^{\sqrt{3}} = 2[9\sqrt{3} - 3\sqrt{3}] = 12\sqrt{3}.$$

121. If $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$, then the quadratic equation whose roots are $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ is

- (A) $x^2 - 6x + 9 = 0$ (B) $x^2 - 7x + 8 = 0$
(C) $x^2 - 14x + 49 = 0$ (D) $x^2 - 10x + 21 = 0$

Correct Option: D

Solution:

$$\text{Since, } f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$$

$$\text{Now, } \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^2 - 1)$$

$$= \lim_{h \rightarrow 0} [(2 - h)^2 - 1] = \lim_{h \rightarrow 0} [4 + h^2 - 4h - 1]$$

$$= \lim_{h \rightarrow 0} [h^2 - 4h + 3] = 0 - 0 + 3 = 3$$

$$\text{and } \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (2x + 3)$$

$$= \lim_{h \rightarrow 0} [2(2 + h) + 3] = \lim_{h \rightarrow 0} [4 + 2h + 3] = 7$$

Hence the quadratic equation whose roots are 3 and 7 is given by $x^2 - (3 + 7)x + (3 + 7) = 0 \Rightarrow x^2 - 10x + 21 = 0$

122. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k}$ intersect at a point, then the integer k is equal to

- (A) -5 (B) 5 (C) 2 (D) -2

Correct Option: A

Solution:

When the two lines intersect then shortest distance between them is zero i.e.

$$\frac{(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} = 0$$

$$\Rightarrow (\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2 = 0$$

$$\text{where } \vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_1 = k\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b}_2 = 3\hat{i} + k\hat{j} + 2\hat{k}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$$\Rightarrow 1(4 - 3k) - 1(2k - 9) - 2(k^2 - 6) = 0$$

$$\Rightarrow -2k^2 - 5k + 25 = 0 \Rightarrow k = -5 \text{ or } \frac{5}{2}$$

123. If the co-ordinates of the point in which the line joining the points (3, 5, -7) and (-2, 1, 8) is intersected by the plane yz is $[a, \frac{13}{b}, c]$, then $(a + b - c) =$

- (A) 2 (B) 3 (C) 4 (D) 5

Correct Option: B

Solution:

$$\text{Ratio} = -\left(\frac{3}{-2}\right) = \frac{3}{2}$$

$\therefore$ Required co-ordinates of the points are

$$\left[\frac{6-6}{5}, \frac{10+3}{5}, \frac{-14+24}{5}\right] = \left(0, \frac{13}{5}, 2\right).$$

$$\therefore a + b - c = 0 + 5 - 2 = 3$$

124. The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at $x = 1$ is:

- (A) $\frac{1}{2}$ (B) 0
(C) -2 (D) ∞

Correct Option: B

Solution:

$$\text{Given curve is } x = 3t^2 + 1 \quad \dots(i)$$



$$\therefore \frac{dx}{dt} = 6t$$

Second curve is $y = t^3 - 1$... (ii)

$$\therefore \frac{dy}{dt} = 3t^2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 3t^2 \times \frac{1}{6t} = \frac{t}{2}$$

But from (i) when $x = 1$

$$\text{we have } 1 = 3t^2 + 1 \Rightarrow 3t^2 = 0 \Rightarrow t = 0$$

$$\therefore \text{When } x = 1 \text{ then } t = 0 \quad \therefore \frac{dy}{dx} = 0$$

Hence, slope of the tangent to the curve = 0

125. A vector perpendicular to the plane containing the vectors $\hat{i} - 2\hat{j} - \hat{k}$ and $3\hat{i} - 2\hat{j} - \hat{k}$ is inclined to the vector $\hat{i} + \hat{j} + \hat{k}$ at an angle

- (A) $\tan^{-1} \sqrt{14}$
 (B) $\csc^{-1} \sqrt{14}$
 (C) $\tan^{-1} \sqrt{15}$
 (D) None of these

Correct Option: A

Solution:

A vector perpendicular to the plane is

$$\begin{aligned} & (\hat{i} - 2\hat{j} - \hat{k}) \times (3\hat{i} - 2\hat{j} - \hat{k}) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ 3 & -2 & -1 \end{vmatrix} = -2\hat{j} + 4\hat{k} \\ &\Rightarrow \text{unit vector } \hat{a} = \frac{-2\hat{j} + 4\hat{k}}{\sqrt{4 + 16}} = \frac{-2\hat{j} + 4\hat{k}}{2\sqrt{5}} \end{aligned}$$

Angle between the unit vector and $\vec{r} = \hat{i} + \hat{j} + \hat{k}$

$$= \cos^{-1} \frac{\vec{r} \cdot \hat{a}}{|\vec{r}| \cdot |\hat{a}|} = \cos^{-1} \frac{1}{\sqrt{15}} = \tan^{-1} \sqrt{14}$$

126. Minimum value of $Z = 3x + 5y$ subject to constraints $x + y \geq 2$, $x + 3y \geq 3$, $x, y \geq 0$

- (A) 6 (B) 7 (C) 8 (D) 9

Correct Option: B

Solution:

Z is minimum at $(\frac{3}{2}, \frac{1}{2})$

127. The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are :

- (A) $\frac{p-q}{r-p}, 1$
 (B) $\frac{q-r}{p-q}, 1$
 (C) $\frac{r-p}{p-q}, 1$
 (D) None of these

Correct Option: C

Solution:

Given equation is

$$(p - q)x^2 + (q - r)x + (r - p) = 0$$

By using formula for finding the roots viz: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we get

$$\begin{aligned} x &= \frac{(r - q) \pm \sqrt{(q - r)^2 - 4(p - q)(p - q)}}{2(p - q)} \\ \Rightarrow x &= \frac{(r - q) \pm (q + r - 2p)}{2(p - q)} = \frac{r - p}{p - q}, 1 \end{aligned}$$

128. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits that the odd digits occupy even positions

- (A) 16 (B) 36 (C) 60 (D) 100

Correct Option: C

Solution:

Here we have 4 odd digits (3, 3, 5, 5) and 5 even digits (2, 2, 8, 8, 8).

$OEOEOEOEO$

where, $E \rightarrow$ even place and $O \rightarrow$ odd place

$\Rightarrow$ Number of ways odd digits will be placed on even places

$$= \frac{4!}{2!2!}$$

Number of ways even digits will be placed on odd places

$$= \frac{5!}{2!3!}$$

$$\text{Therefore, total number of ways} = \frac{4!}{2!2!} \times \frac{5!}{2!3!} = 60$$

129. The negation of the Boolean expression $((\sim q) \wedge p) = ((\sim p) \vee q)$ is logically equivalent to

- (A) $p \Rightarrow q$ (B) $q \Rightarrow p$
 (C) $\sim (p \Rightarrow q)$ (D) $\sim (q \Rightarrow p)$

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Correct Option: C**Solution:**

$$\sim p \vee q \equiv p \rightarrow q$$

$$\sim q \wedge p \equiv \sim (p \rightarrow q)$$

Negation of $\sim (p \rightarrow q) \rightarrow (p \rightarrow q)$ is $\sim (p \rightarrow q) \wedge \sim (p \rightarrow q)$ i.e.

$$\sim (p \rightarrow q)$$

130. General solution of $\tan 5\theta = \cot 2\theta$ is (where $n \in \mathbb{I}$) :-

$$(A) \theta = \frac{n\pi}{7} + \frac{\pi}{14}$$

$$(B) \theta = \frac{n\pi}{7} + \frac{\pi}{5}$$

$$(C) \theta = \frac{n\pi}{7} + \frac{\pi}{2}$$

$$(D) \theta = \frac{n\pi}{7} + \frac{\pi}{3}$$

Correct Option: A**Solution:**

$$\tan 5\theta = \cot 2\theta \Rightarrow \frac{\sin 5\theta}{\cos 5\theta} - \frac{\cos 2\theta}{\sin 2\theta} = 0$$

$$\Rightarrow \frac{\sin 5\theta \sin 2\theta - \cos 5\theta \cos 2\theta}{\cos 5\theta \sin 2\theta} = 0$$

$$\Rightarrow -\cos(5\theta + 2\theta) = 0 \Rightarrow \cos 7\theta = 0$$

$$\Rightarrow 7\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{14} = \frac{n\pi}{7} + \frac{\pi}{14}$$

Hence result

OR

$$\Rightarrow \tan 5\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow 5\theta = n\pi + \frac{\pi}{2} - 2\theta$$

$$\Rightarrow 7\theta = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{n\pi}{7} + \frac{\pi}{14}$$

BITSAT Solved Paper 2023 Session-II - ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	D	A	D	D	C	C	B	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	D	B	C	C	C	A	D	D
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	B	D	D	A	B	A	C	B
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	A	B	D	D	A	B	D	A	A	A
Que.	41	42	43	44	45	46	47	48	49	50
Ans.	B	D	A	B	A	A	A	C	A	D
Que.	51	52	53	54	55	56	57	58	59	60
Ans.	C	A	B	D	B	C	B	C	D	D
Que.	61	62	63	64	65	66	67	68	69	70
Ans.	D	A	B	D	A	B	A	D	A	A
Que.	71	72	73	74	75	76	77	78	79	80
Ans.	B	D	C	C	A	A	A	C	D	A
Que.	81	82	83	84	85	86	87	88	89	90
Ans.	C	D	A	C	D	D	C	A	A	D
Que.	91	92	93	94	95	96	97	98	99	100
Ans.	A	A	D	C	D	B	C	C	B	A
Que.	101	102	103	104	105	106	107	108	109	110
Ans.	B	A	C	B	B	D	C	B	C	D
Que.	111	112	113	114	115	116	117	118	119	120
Ans.	A	A	A	D	A	C	A	C	D	B
Que.	121	122	123	124	125	126	127	128	129	130
Ans.	D	A	B	B	A	B	C	C	C	A